

# Decoupling or nondecoupling: is that the $R_b$ question?

D. Comelli<sup>a</sup> and João P. Silva<sup>b</sup>

<sup>a</sup> Departament de Física Teòrica and IFIC  
Universitat València – CSIC  
E-46100 Burjassot (València), Spain

<sup>b</sup> Centro de Física Nuclear da Univ. de Lisboa  
Av. Prof. Gama Pinto, 2  
1699 Lisboa Codex, Portugal  
and  
Centro de Física  
Instituto Superior de Engenharia de Lisboa.

February 1, 2008

## Abstract

The top quark is well known for the nondecoupling effects it implies in  $\rho$  and  $R_b$ . The recent experimental  $R_b$  data exhibits a disagreement with the SM prediction at more than the  $3\sigma$  level. It is tempting to explore whether this might be due to nondecoupling New Physics effects, opposite to those of the top. We investigate this issue in the context of models with an extra family of right or left handed, singlet or doublet quarks. It is shown that, contrary to what one might naively expect, the nondecoupling properties of a mirror  $t'$  do not have an impact on  $R_b$ , due to a conspiracy of the mixing angles, imposed by the requirement that there be no  $b - b'$  mixing. Our analysis agrees with an analysis performed independently, which includes this model as a particular case.

# 1 Introduction

The high precision experiments on electroweak observables have yielded spectacular confirmations of the Standard Model, including in the structure of radiative corrections. In particular, the experiments performed on the  $Z$  resonance at LEP have probed the couplings of the  $Z$  to leptons and quarks. Here, LEP has found significant deviations from the SM predictions on the observed ratios  $R_b = \Gamma_b/\Gamma_{had}$  and  $R_c = \Gamma_c/\Gamma_{had}$  [1, 2]. The former presents a  $3.7\sigma$  deviation from the SM value ( $R_b^{SM} = 0.2156$  for a mass of the top  $m_t = 174$  GeV), if  $R_c$  is used as a free parameter to fit the data. Conversely, one finds  $R_b = 0.2205 \pm 0.0017$ , that is  $3\sigma$  away from the SM prediction, if  $R_c$  is fixed at its SM value. This discrepancy might be the first window into Physics beyond the SM. Since  $R_b$  exhibits at present a bigger deviation, we shall concentrate on it. There are several features that make this decay special: 1) since the bottom is the isospin partner of a ‘heavy’ quark, and scalar-fermion couplings are typically proportional to fermion masses, new scalars might give here relevant contributions; 2) the hierarchical structure of the CKM matrix indicates that any heavier family might couple mostly to the  $b$  and  $t$  quark; 3) the  $Z \rightarrow b\bar{b}$  decay is well known for the nondecoupling loop contributions that it gets from the top quark [3]. It turns out that, if one ignored these contributions (in a blatantly unphysical fashion), the result would be  $R_b(m_t = 0) = 0.220$  [4], in accordance with experiment!

Recently, there have been many models proposed to solve these discrepancies, highlighting the first two points. Some solutions have been sought within well motivated theories like Extended Technicolour [5], although the solutions are quite contrived, and Supersymmetry [6, 7]. In the later, the modifications arise through radiative corrections<sup>1</sup>, but the relevant parameter space almost disappears in light of the recent “LEP 1.5” run [7, 9].

There has also been a large number of phenomenological proposals. In general, we can write the tree level coupling of  $Z$  with a fermion  $f$  as

$$\mathcal{L}_Z = \frac{g}{c_W} Z_\mu \left[ g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R \right] , \quad (1)$$

with

$$g_\alpha^f = T_3(f_\alpha) - Q(f)s_W^2 , \quad \alpha = L, R , \quad (2)$$

in the absence of isospin mixing. This has prompted phenomenological solutions of both problems where one allows for tree level mixings of the  $b$  and/or  $c$  quarks with new quarks of the same charge and different weak isospin [10, 11, 12]. Another possibility arises with the introduction of a new “hadrophilic”  $Z'$  that mixes with the  $Z$  [13, 14, 15]. For a given choice of parameters, this also allows for an explanation of the excess of dijet events observed at CDF. We note that, in some of the models above one should also worry about the implications on the oblique radiative corrections.

In this article we stress that the third feature mentioned above might be the key to the puzzle. Namely, that the disagreement found in  $R_b$  might be signaling us the

---

<sup>1</sup>The simple two Higgs doublet model also contributes radiatively to  $R_b$ , but it only provides a solution when the pseudoscalar mass is not much larger than 50 GeV, and  $\tan\beta > 70$  [8].

existence of New Physics through its nondecoupling effects. The top quark, recently discovered by CDF [16] and D0 [17], exhibits nondecoupling effect in both  $\rho$  and  $R_b$ . When the experimental data of the later is confronted with the SM values, one finds that the radiative corrections (RC) push the SM in the wrong direction, worsening the problem proportionally to  $m_t^2$  [3].

Here, we will investigate whether this discrepancy might be explained using the same nondecoupling mechanism that is at work in the SM but with the opposite sign. We do so in the context of models with a fourth family of right or left handed, singlet or doublet quarks. The mirror quark model stands out as a leading candidate, since a mirror  $t'$  with fixed mixing angle would indeed produce a nondecoupling effect with the correct sign. However, if one requires that there be no tree level  $b - b'$  mixing, a conspiracy of the mixing angles cancels this effect. In the next section we will briefly discuss nondecoupling effects. We then turn to the addition of mirror quarks to the SM and investigate its consequences. We finish by presenting our conclusions.

## 2 Decoupling and nondecoupling

As is well known, the decoupling theorem states that the physical effects of a heavy particle are suppressed at low energies by the inverse powers of the heavy mass scale, if all the other parameters are held fixed [18]. An exception to this theorem occurs naturally in gauge theories with spontaneous symmetry breaking. Here, the fermions and scalars often get mass through their Yukawa couplings with Higgs fields. When these Higgs fields get (fixed) vacuum expectation values, we can only increase the mass of those particles increasing the respective Yukawa coupling. These large Yukawa couplings may entail a violation of the decoupling theorem whenever they compensate the heavy mass suppression arising from the propagator [19]. In the SM the leading RC to the  $Zb\bar{b}$  vertex come, in the Feynman Gauge, from diagrams involving top quarks and charged would-be-Goldstone bosons. These couple very strongly to the fermionic line, for they are proportional to the Yukawa coupling of the top  $y_t$ . However, this fact is not sufficient to get a nondecoupling contribution.

In a very elegant article, Liu and Ng [20] have stressed that nondecoupling corrections to the  $Zb\bar{b}$  vertex only occur if the fermion present in the loop transforms chirally under  $SU(2) \times U(1)$ , and if its mass is large compared to that of the exchanged boson. It has become standard to parameterize both the top and New Physics' impact on the bottom vertex by [20, 21]

$$g_L^b \rightarrow g_L^b + \delta g_L^b \quad , \quad g_R^b \rightarrow g_R^b + \delta g_R^b . \quad (3)$$

These changes are small if one has small tree-level mixing angles or loop correction. A recent fit to the electroweak observables yields [21]

$$\delta g_L^b = -0.0033 \pm 0.0035 \quad , \quad \delta g_R^b = +0.018 \pm 0.013 , \quad (4)$$

with a large correlation between these parameters. To first order, the change in  $R_b$  will then be proportional to  $g_L^b \delta g_L^b + g_R^b \delta g_R^b$ . Thereby, a positive  $\delta g_L^b$  change will reduce

$R_b$ , worsening the problem one already has without such contributions<sup>2</sup>. A similar situation occurs for a negative  $\delta g_R^b$  change.

We follow Liu and Ng, and take an interaction of the form  $y_F \bar{b}_L \phi^- F_R + \text{h.c.}$ , where  $\phi$  is the would-be-Goldstone boson,  $F_R$  an ordinary or new right handed quark whose left handed partner,  $F_L$ , may transform differently under  $SU(2)$ . Obviously, neglecting  $m_b$ , one only changes the left handed coupling. In the  $m_F \gg M_Z$  limit, we find

$$\delta g_L^b = \frac{\alpha}{4\pi s_W^2} \left( \frac{y_F}{g} \right)^2 [T_3(F_R) - T_3(F_L)] \Delta_0\left(\frac{m_F^2}{M_Z^2}\right) + \mathcal{O}\left(\frac{M_Z^2}{m_F^2}\right) \quad (5)$$

where  $\Delta_0(x) = \frac{x}{1-x} + \frac{x}{(1-x)^2} \log x$  (which lies between 0 and  $-1$ ), confirming the results of ref. [20]. In the case of the top quark ( $T_3(t_R) = 0$  and  $T_3(t_L) = 1/2$ ) one finds the well known  $m_t^2$  dependence [3],

$$\delta g_L^{b \text{ SM}} \propto \left( \frac{y_t}{g} \right)^2 \left[ -\frac{1}{2} \right] (-1) = +\frac{m_t^2}{4M_W^2}. \quad (6)$$

The fact that this change is positive shows that the nondecoupling one loop contribution due to the top reduces  $R_b$ , worsening the problem considerably, and leading to the final  $3.7\sigma$  deviation from experiment.

In the case of a new vector-like top' (singlet or doublet),  $T_3(F_R) = T_3(F_L)$  and the induced RC are subleading with respect to the SM one,

$$\delta g_L^{b \text{ vectorlike}} \propto \mathcal{O}\left(\frac{M_Z^2}{m_{t'}^2}\right) \quad (7)$$

However, a very interesting situation occurs in the presence of mirror fermions. In that case, the isospin quantum numbers of the  $t'$  are  $T_3(t'_R) = 1/2$  and  $T_3(t'_L) = 0$ , generating a contribution of the form

$$\delta g_L^{b \text{ mirror}} \propto -\frac{m_{t'}^2}{4M_W^2}. \quad (8)$$

which is nondecoupling like the top, but appears naturally with the opposite sign. Notice that, as in the case of the SM, the  $\gamma b \bar{b}$  vertex is protected from non decoupling contributions by the Ward identities, as can be checked explicitly performing the substitutions  $g/2c_w(T_3 - 2s_w^2 Q) \rightarrow eQ$  and  $T_3 \rightarrow 0$ . These (far too) simple considerations would lead one to believe that the addition to the SM of a fourth family of mirror fermions, in which  $t'$  mixes with  $t$ , might entail a simple and natural phenomenological solution of the  $R_b$  puzzle. We shall prove in the next section that this is not so and explain how that arises as a consequence of the requirement that there be no tree level  $b - b'$  mixing.

---

<sup>2</sup> If  $\delta g_R^b = 0$  one recovers the case discussed in ref. [22], with  $\delta_{b\text{-vertex}} = 2\delta g_L^b$ , and in ref. [23], with  $\epsilon_b = -2\text{Re}(\delta g_L^b)$ .

### 3 The decoupling of mirror fermions

In this section we present the main features of a model with an extra fourth mirror family. We will concentrate only on the quark sector. The extra leptons (present only in order to cancel the anomaly) are assumed to decouple completely from the SM ones. We will follow the notation of Lavoura and Silva [24, 25] and parameterize the mixing of singlet and doublet quarks with mixing matrices  $V_L$  and  $V_R$  in terms of which the charged boson interactions become

$$\frac{g}{\sqrt{2}} W_\mu^\dagger \bar{u}_i \gamma^\mu [V_L \gamma_L + V_R \gamma_R]_{ij} d_j + \text{H.c.} , \quad (9)$$

where,  $i$  ( $j$ ) runs over the number of charge  $2/3$  ( $-1/3$ ) quarks, and  $\gamma_{R,L} = (1 \pm \gamma_5)/2$ . Notice that the matrix  $V_R$  will exist whenever the right handed quarks belong to nontrivial multiplets, since the gauge bosons couple with fermions through the weak isospin. In general, these matrices are not unitary but are part of larger unitary matrices [24]. The neutral current interaction in the presence of mixing becomes,

$$\frac{g}{2c_w} Z_\mu \left[ \bar{u}_i \gamma^\mu \left( -\frac{4}{3} s_w^2 + U^L \gamma_L + U^R \gamma_R \right)_{ij} u_j + \bar{d}_i \gamma^\mu \left( \frac{2}{3} s_w^2 - D^L \gamma_L - D^R \gamma_R \right)_{ij} d_j \right] , \quad (10)$$

where the hermitian mixing matrices  $D$  and  $U$  represent projection operators and are given by ( $\alpha = L, R$ )

$$U^\alpha = V_\alpha V_\alpha^\dagger, \quad , \quad D^\alpha = V_\alpha^\dagger V_\alpha . \quad (11)$$

It is easy to see that the effective weak isospin of  $d_{jL}$  is then given by the  $jj$  component of  $-D^L/2$ , and similarly for the others. Thus, in  $Z \rightarrow b\bar{b}$ , one is probing

$$\delta g_L^b = (1 - D_{bb}^L)/2 \quad , \quad \delta g_R^b = -D_{bb}^R/2. \quad (12)$$

Comparing Eqs. (4) and (12) we see that the tree level mixing of the  $b$  with a  $b'$ , which can be either singlet or (lower component of) doublet in either the left or right hand, will always deepen the problem. Therefore, one must impose  $D_{bb}^L = 1$  and  $D_{bb}^R = 0$ . Of course, by looking at Eq. (2) one can easily understand this result and see what isospin assignments must exist in order to circumvent it [12].

In what follows we will take a fourth family of mirror quarks, assuming that the hierarchical structure of the CKM matrix is preserved, so that the mixing of the new  $t'$  will occur predominantly<sup>3</sup> with  $t$ . Due to the absence of  $b$  mixing, the matrices have a very simple form [25]

$$V_L = \begin{bmatrix} c_1 & 0 \\ s_1 & 0 \end{bmatrix} \quad V_R = \begin{bmatrix} 0 & c_3 \\ 0 & s_3 \end{bmatrix} \quad U^L = \begin{bmatrix} c_1^2 & c_1 s_1 \\ c_1 s_1 & s_1^2 \end{bmatrix} \quad U^R = \begin{bmatrix} c_3^2 & c_3 s_3 \\ c_3 s_3 & s_3^2 \end{bmatrix} . \quad (13)$$

Here, the angles  $\theta_1$  and  $\theta_3$  are those mixing the left and right handed  $Q = 2/3$  quarks when one goes from the weak basis into the mass basis, through

$$\begin{bmatrix} \frac{v}{\sqrt{2}} \Delta & M_q \\ M_p & \frac{v}{\sqrt{2}} \Sigma \end{bmatrix} = \begin{bmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{bmatrix} \begin{bmatrix} m_t & 0 \\ 0 & m_{t'} \end{bmatrix} \begin{bmatrix} s_3 & c_3 \\ -c_3 & s_3 \end{bmatrix} . \quad (14)$$

---

<sup>3</sup> One might also tackle the experimental  $R_c$  results by mixing the  $c$  quark with  $t$  and  $t'$ . In such a context, the hierarchy of the CKM matrix might also have an interpretation. We shall not do that here.

Therefore,

$$M_q = c_1 c_3 m_t + s_1 s_3 m_{t'} = 0, \quad (15)$$

$$\frac{v}{\sqrt{2}} \Delta = c_1 s_3 m_t - s_1 c_3 m_{t'}, \quad (16)$$

where the vectorlike doublet mass term  $M_q$ , that is built with the left handed third family doublet and the right handed fourth family doublet, must be zero since we require that  $b$  and  $b'$  do not mix. This constraint equation will be crucial in deriving the decoupling properties. Also, since  $\Delta$  is the mass term of the upper component of the doublet containing  $b_L$ , the would-be-Goldstone boson couples  $b_L$  to the physical quarks, proportionally to it.

In the  $m_b \simeq 0$  limit, the Lagrangian describing the interaction with the charged would-be-Goldstone bosons becomes

$$\frac{\sqrt{2}\phi^-}{v} [\bar{u}_i X_{ib} \gamma_L b] + \text{H.c.}, \quad (17)$$

where

$$X = (1 - U^R) \text{diag}(m_t, m_{t'}) V_L = \frac{v}{\sqrt{2}} \Delta \begin{bmatrix} s_3 & 0 \\ -c_3 & 0 \end{bmatrix}. \quad (18)$$

If there is no  $t - t'$  mixing,  $s_1 = 0 = c_3$ ,  $V_L = 1$ , and  $U^R = 0$  recovering the SM result. Notice the proportionality to  $\Delta$ , as we had anticipated.

The total leading contribution to  $\delta g_L^b$ , including also the top, is

$$\begin{aligned} \frac{8\pi s_w^2 M_w^2}{\alpha} \delta g_L^b &= X_{ib}^* X_{jb} \left\{ \left[ U^R/2 - U^L/2 \right]_{ij} m_i m_j C_0(q^2, m_W, m_i, m_j) \right. \\ &\quad \left. \left[ U^R/2 - Q(t) s_w^2 \right]_{ij} \rho_3(q^2, m_W, m_i, m_j) \right\} \\ &+ |X_{ib}|^2 \left[ T_3(\phi^-) - Q(\phi^-) s_W^2 \right] \rho_4(q^2, m_W, m_i, m_j), \end{aligned} \quad (19)$$

where the repeated indices  $i, j$  are summed from  $t$  to  $t'$ , and the functions which appear are defined in ref. [26]. In deriving this result we have used the trivial fact that  $U_R X = 0$ . This generalizes the results in refs. [20, 26], for this case in which there is isospin mixing. For  $m_{t,t'} > M_w$  the  $\rho_{3,4}$  contributions are subleading and the nondecoupling  $C_0$  term yields

$$\begin{aligned} \delta g_L^b &\propto -\frac{(c_1 s_3 m_t - c_3 s_1 m_{t'})^2}{4M_w^2} \left[ (c_1^2 - c_3^2)(c_3^2 - s_3^2) \right. \\ &\quad \left. + 2s_3 c_3 (c_1 s_1 - c_3 s_3) \frac{m_t m_{t'}}{m_{t'}^2 - m_t^2} \log \frac{m_{t'}^2}{m_t^2} \right] \end{aligned} \quad (20)$$

As expected, the final expression is proportional to  $\Delta^2$ . Fixing  $m_t = 175$  GeV would leave three parameters, were it not for the constraint  $m_t c_1 c_3 + m_{t'} s_1 s_3 = 0$ , imposed by Eq. (15). Thus  $\delta g_L^b$  is a function of only two parameters, for example, the mixing angles  $s_1$  and  $c_3$ . The SM result, cf. Eq. (6), is correctly reproduced in the limit  $s_1 = 0 = c_3$ .

Let us look at the impact of Eq. (15) more closely. It clearly implies that  $\theta_1$  and  $\theta_3$  must lie in adjacent quadrants. Using only this fact, one can show that the coefficient within the squared brackets is always larger than  $-1$ . In addition, it can only exceed 0 marginally, achieving around  $+0.125$  when  $c_1$  is close to one and  $c_3$  is close to 0.85. This is easily understood. In fact, taking  $m_{t'}$  to infinity kills the logarithm, leaving the first term. This will be positive when  $c_1^2 > c_3^2 > s_3^2$ . For *fixed* mixing angles, the overall result would then exhibit nondecoupling with the correct (negative) sign, solving the  $R_b$  puzzle. Unfortunately, Eq. (15) does more than fix the relative signs. It also implies a relation between the parameters, which we may choose to write as

$$s_3^2 = \left[ \left( \frac{m_{t'} s_1}{m_t c_1} \right)^2 + 1 \right]^{-1} . \quad (21)$$

For fixed  $c_1$ , this ends up changing the sign of the decoupling, as we increase  $m_{t'}$ . The limit of  $c_1 \geq c_3 \rightarrow 1$  must be taken carefully and we obtain the SM result!

Another interesting limit arises for  $m_t = m_{t'}$ . In this case Eq. (15) implies  $s_3 = -c_1$  and  $c_3 = s_1$ , and we recover again the SM result. In fact, any such mixing is allowed, as we can see by looking back at Eq. (14). For this case, the mass matrix was already diagonal and proportional to unity in the weak basis.

Expressing everything in terms of  $m_{t'}/m_t > 1$  and  $c_1$ , one can show that Eq. (20) may, at best, reproduce the SM<sup>4</sup>. This occurs whenever  $c_1 = 1$ , and also for any case with  $m_t = m_{t'}$ . With hindsight, this is a simple consequence of the fact that the  $b$  vertex picks up those particles that couple primarily to it, while  $t'$ , whose decoupling one would wish to use, couples primarily to  $b'$ . It is tempting to conjecture that such a situation will occur in any model where this trick is attempted. Prudence advises that one should wait before any strong claim is made [27].

Three possibilities to evade this conclusion come immediately to mind. One may take the  $t'$  as the quark produced at the Tevatron and have a lighter  $t$  [28, 29]. This reduces the  $m_t^2$  prefactor ameliorating the problem. Other possibilities arise taking  $m_{t'}/m_t < 1$ . Either  $t$  is produced at the Tevatron and  $t'$  is lighter than 175 GeV, or the  $t'$  is the one produced at the Tevatron and  $t$  is heavier than 175 GeV. These options already have strong experimental constraints from direct searches, and we shall not discuss them further except to point out other nondecoupling properties that must be faced.

In fact, one must also worry about the nondecoupling effects present in the oblique radiative corrections [30, 31]. We adopt the  $S$  and  $T$  parameterization of Peskin and Takeuchi, for which the last reported constraints are within  $1\sigma$  of the SM, but tending towards negative values [2]. Any violation of the custodial symmetry through mass splittings among particles inside a multiplet will have an impact on  $T$ . In turn, the  $S$  parameter is sensitive to the chiral breaking. Their expressions for extensions of the SM with the addition of an arbitrary number of vectorlike or mirror fermions are given in ref. [25]. For the simple case in which  $m_t = m_{t'}$ , one finds

$$T = T^{\text{SM}}(m_t, m_b) + T^{\text{SM}}(m_t, m_{b'}) ,$$

---

<sup>4</sup>There is a region for  $c_1 \approx 1$  and  $m_{t'}/m_t > 10$ , where the new contribution may cancel, or even exceed the one from the top. However, this corresponds to values of the Yukawa couplings  $\Delta$  and  $\Sigma$ , way beyond the perturbative regime.

$$S = S^{\text{SM}}(m_t, m_b) + S^{\text{SM}}(m_t, m_{b'}) , \quad (22)$$

where the first term is the SM contribution and the second has the same functional dependence but with  $m_b$  substituted by  $m_{b'}$ . This illustrates in a simple way that  $m_{b'}$  cannot be much larger than  $m_{t'}$  for  $T$  grows with the difference of the squared masses. On the other hand, a fourth family of degenerate fermions yields an additional  $2/(3\pi)$  contribution to  $S$  and is allowed at the 95% level [2]. In strict model building, one might introduce particles in higher multiplets [32] to reduce  $S$ , but that lies outside the scope of this article.

## 4 Conclusions

Prompted by the  $R_b$  puzzle, we have analyzed the decoupling properties of an extra  $t'$  quark that runs in the dominant  $Z \rightarrow b\bar{b}$  vertex correction. This is done in the context of a model with extra left or right handed, singlet or doublet quarks. It is well known that a sequential family produces effects that go in the wrong direction. Vectorlike effects are subdominant.

We point out that a mirror  $t'$  with fixed mixing angle would exhibit nondecoupling with the correct sign, apparently solving the problem. However, when one imposes the absence of tree level  $b - b'$  mixing (that would take  $R_b$  in the wrong direction), the mixing angles conspire to destroy those nondecoupling effects.

It remains to be seen whether complete models may be built with such nondecoupling New Physics effects. That would be a very elegant solution to the  $R_b$  puzzle. It would be similar to the situation that occurred when one knew that the top quark had to exist due to its nondecoupling effects in  $\rho$ , prior to its discovery by CDF [16] and D0 [17]. Our study of the decoupling properties of the mirror  $t'$  shows the importance of the mixing angles in extracting such conclusions, and selecting viable models.

**Final Note:** After this work was completed we received a comprehensive independent analysis by Bamert, Burgess, Cline, London, and Nardi [33]. Their analysis includes this model as a particular case and we agree with their results. Here we have chosen to concentrate on the nondecoupling properties from the start and, hence, include the crucial constraint Eq. (15) at the end. Since this constraint is common to a large class of models, they have used it directly in the initial Lagrangian. This corresponds to noting that the  $ib$  components of  $U^R \text{diag}(m_t, m_{t'})V_L$  are proportional to  $M_q$ , so that, if one uses Eq. (15) directly on the Lagrangian of Eq. (18) one reproduces their Eq. (48) [34].

## Acknowledgments

We would like to thank A. Barroso and L. Lavoura for useful advice. J. P. S. is indebted to A. Eiró for the facilities provided, and to L.-F. Li for constantly stressing the importance of the decoupling properties. The work of D. C. was funded by the



Spanish Comisión Interministerial de Ciencia y Tecnología. We are greatly indebted to the authors of reference [33], who have graciously pointed out to us a wrong statement we had originally made about their article.

## References

- [1] Recent reviews may be found in W. Hollik, talk presented at the International Eu-  
rophysics Conference on High Energy Physics, hep-ph/9512232 (1995); P. Bamert,  
McGill University preprint McGill/95-64.
- [2] P. Langacker, talk presented at SUSY-95, hep-ph/9511207 (1995).
- [3] A. Akundov, D. Bardin, and T. Riemann, Nucl. Phys. **B276**, 1 (1986); J.  
Bernabéu, A. Pich, and A. Santamaria, Phys. Lett. **B200**, 569 (1988); W.  
Beenakker, W. Hollik, Z. Phys. C **40**, 141 (1988).
- [4] See, for example, J. D. Wells, and G. L. Kane, hep-ph/9510372 (1995).
- [5] N. Kitazawa, Phys. Lett. **B313**, 395 (1993); R. S. Chivukula, E. H. Simmons,  
and J. Terning, Phys. Lett. **B331**, 383 (1994); C. T. Hill and X. Zhang, Fermilab  
preprint Pub/94-231 (1994).
- [6] M. Bouleware and D. Finnel, Phys. Rev. D **44**, 2054 (1991). A. Djouadi, G.  
Girardi, W. Hollik, F. M. Renard, and C. Verzegnassi, Nucl. Phys. **B349**, 48  
(1991). G. Altareli, R. Barbieri, and F. Caravaglios, Nucl. Phys. **B405**, 3 (1993);  
*ibid* Phys. Lett. **B314**, 357 (1993); C. S. Lee, B. K. Hou, J. M. Yang and Z. Y.  
Fang, J. Phys. G **19**, 13 (1993); J. D Wells, C. Kolda, and G. L. Kane, Phys.  
Lett. **B338**, 219 (1994); D. Garcia, R. Jiménez, and J. Solà, Phys. Lett. **B347**,  
321 (1995); G. Kane, R. Stuart, and J. Wells, Phys. Lett. **B354**, 350 (1995); D.  
Garcia and J. Solà, Phys. Lett. **B357**, 349 (1995); X. Wang, J. L. Lopez, and D.  
V. Nanopoulos, Phys. Rev. D **52**, 4116 (1995); A. Brignole, F. Feruglio, and F.  
Zwirner, CERN preprint CERN-TH-95-340.
- [7] E. H. Simmons, and Y. Su, hep-ph/9602267 (1996).
- [8] A. Djouadi, J. L. Kneur, and G. Moultaka Phys. Lett. **B242**, 265 (1990); A.  
Denner, R. J. Guth, W. Hollik, and J. H. Kuhn, Z. Phys. C **51**, 695 (19).
- [9] J. Ellis, J. L. Lopez, and D. V. Nanopoulos, CERN preprint CERN-TH/95-314  
(1995).
- [10] G. Bhattacharyya, G. C. Branco, and W.-S. Hou, CERN preprint CERN-TH/95-  
326 (1995).
- [11] E. Ma, University of California at Riverside preprint UCRHEP-T153 (1995).

- [12] C.-H. V. Chang, D. Chang and W.-Y. Keung, National Tsing-Hua University preprint NHCU-HEP-96-1 (1996).
- [13] B. Holdom, Phys. Lett. **B339**, 114 (1994), and Phys. Lett. **B351**, 279 (1995).
- [14] P. Chiappetta, J. Layssac, F. M. Renard, and C. Verzegnassi, hep-ph/9601306 (1996).
- [15] G. Altarelli, N. Di Bartolomeo, F. Feruglio, R. Gatto, and M. L. Mangano, CERN preprint CERN-TH/96-20 (1996).
- [16] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **74**, 2626 (1995).
- [17] D0 Collaboration, S. Abachi *et al.*, Phys. Rev. Lett. **74**, 2632 (1995).
- [18] T. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975); K. Szymanzik, Comm. Math. Phys. **34**, 7 (1973).
- [19] M. Veltman, Nucl. Phys. **B123**, 89 (1977); D. Collins, F. Wilczek, and A. Zee, Phys. Rev. D **18**, 242 (1978); D. Toussiant, Phys. Rev. D **18** 1626 (1978); L. H. Chan, T. Hagiwara, and B. Ovrut, Phys. Rev. D **20** 1982 (1979); L.-F. Li and T. P. Cheng, Carnegie Mellon University preprint CMU-HEP-91-11, published in DPF Conf.91 proceedings, Vancouver, Canada (1991).
- [20] J. T. Liu and D. Ng, Phys. Lett. **B342**, 262 (1995).
- [21] J. Erler and P. Langacker, Phys. Rev. D **52**, 441 (1995); P. Langacker in ref. [2].
- [22] J. Bernab  u, A. Pich, and A. Santamaria, Nucl. Phys. **B363**, 326 (1991).
- [23] G. Altarelli, R. Barbieri, and F. Caravaglios, Phys. Lett. **B349**, 145 (1995). See also their articles in ref. [6].
- [24] L. Lavoura and J. P. Silva, Phys. Rev. D **47**, 1117 (1993).
- [25] L. Lavoura and J. P. Silva, Phys. Rev. D **47**, 2046 (1993).
- [26] A. Denner, R. J. Guth, W. Hollik and J. H. K  hn, Z. Phys. C **51**, 695 (1991).
- [27] Sara Barroso, private communication.
- [28] B. Mukhopadhyaya and S. Nandi, Phys. Rev. Lett. **66**, 285 (1991); Phys. Rev. **D46**, 5098 (1992).
- [29] W. S. Hou and H. C. Huang, Phys. Rev. **D51**, 5285 (1995).
- [30] D. C. Kennedy and B. W. Lynn, Nucl. Phys. **B322**, 1 (1989); D. C. Kennedy and P. Langacker, Phys. Rev. Lett. **65**, 2967 (1990), and **66** 3965(E)(1991), and Phys. Rev. **D44**, 1591 (1991); W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); B. Holdom, Phys. Lett. **B259**, 329 (1991); M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991). B. Holdom and J. Terning, Phys. Lett.

- B247**, 88 (1991); G. Altarelli and R. Barbieri, Phys. Lett. **B253**, 161 (1991); G. Altarelli, R. Barbieri and S. Jadach, Nucl. Phys. **B269**, 3 (1992).
- [31] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990), and Phys. Rev. **D46**, 381 (1992).
- [32] M. J. Dugan and L. Randall Phys. Lett. **B264**, 154 (1991).
- [33] P. Bamert, C. P. Burgess, J. M. Cline, D. London and E. Nardi, McGill University preprint McGill-96/04, hep-ph/9602438 (1996).
- [34] We are indebted to the authors of reference [33], who have kindly pointed this out to us.